A SOLUTION OF EQUATIONS BY STANDARD CURVES.

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If the general quadratic is expressed in the form

$$x^2 - kx + l = 0$$

$a + b = k$ can be represented by a straight line which cuts the

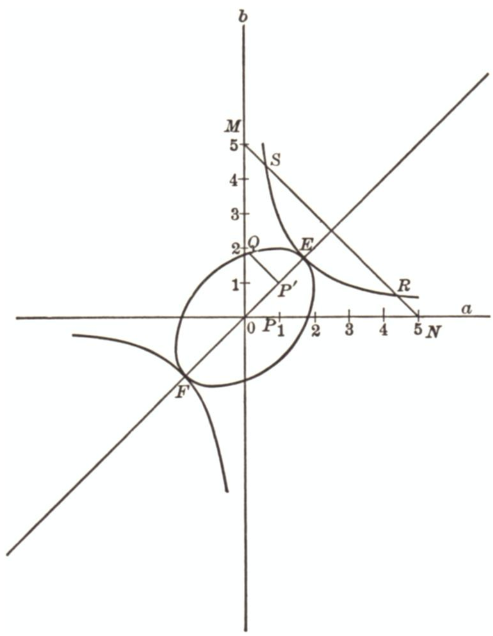

Fig. 1.

Solution: $x^2 - 5x + 3 = 0$, at R, S.
Solution: $x^2 - 2x + 3 = 0$, $OP + P'Qi$.

hyperbola $ab = l$ in two points the co-ordinates of either of which are a and b, the roots of the quadratic.

Example (see Fig. 1):

If $x^2 - 5x + 3 = 0$,

The line $a + b = 5$ cuts the hyperbola $ab = 3$ in the point R, S giving the solution $x = 4.3$ or $.7$.

As the value of $k^2 - 4l$ becomes smaller, l remaining constant, the line MN slides toward the origin, becoming tangent to the hyperbola at $k^2 - 4l = 0$, and not cutting the hyperbola at all when $k^2 - 4l < 0$. The roots of the equation are then complex and are determined by a point on the ellipse whose major axis is the major axis of the hyperbola and whose semi-minor axis equals $5l$. The real part of the complex root is evidently $k/2$, which is measured from the origin along the x axis. At the point thus found, a perpendicular is erected to cut the major

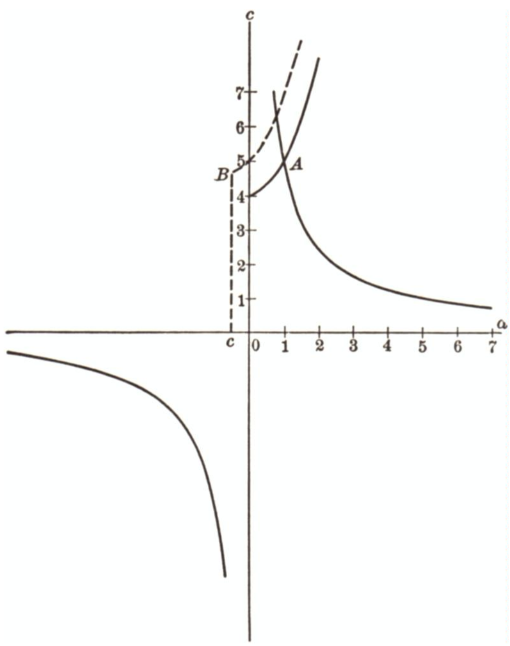

FIG. 2.

Solution: $x^3 + 4x - 5 = 0$, real root at A.
Imaginary roots, $OC \pm i\sqrt{BC}$.

axis of the ellipse. The perpendicular to the major axis from the point thus obtained cuts the ellipse and its length is the coefficient of i in the imaginary part of the root.

Example (Fig. 1):

If $x^2 - 2x + 3 = 0$,
The real part of the root is $OP = 1$;
The imaginary part is $P'Qi = 1.4i$;
The complete root $x = 1 \pm 1.4i$.
In the reduced cubic equation

$$x^3 + px + q = 0.$$

If a is one of the roots and c the product of the other two, then $a^2 - c = -p$ and $ac = -q$. Now $a^2 = c = -p$ is a parabola and $ac = -q$ is an hyperbola, and their points of intersec-

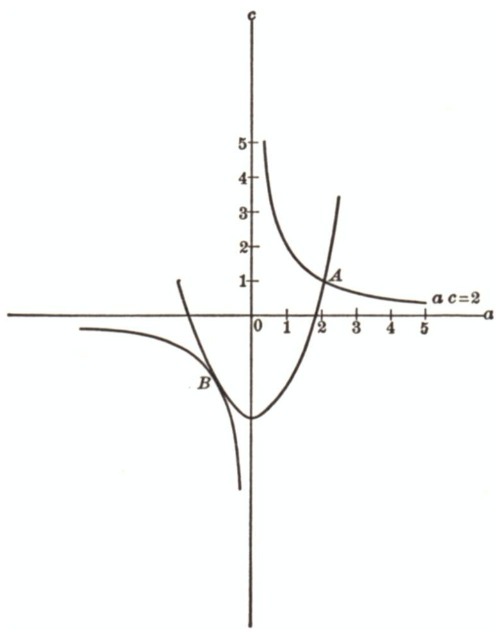

FIG. 3.
Solution: $x^3 - 3x - 2 = 0$. Roots at A and B.

tion will give the values of a and c, and therefore the roots.

Example (Fig. 2):

To solve $x^3 - 3x - 2 = 0$.

$a^2 - c = 3$. Therefore the vertex of the parabola $a^2 - c$ is at -3 on the y axis. It then cuts the hyperbola $ac = 2$ in the

point whose abscissa is 2 and touches the curve at the point whose abscissa is -1. Therefore $a=2$, $c=1$, or $a=-1$, $c=-2$, from which the solution 2, -1, -1 follows at once. If there is only one real root the parabola will cut the hyperbola in only one point; but the quadratic of the other two roots can be written at once and solved by the methods given before, or by the following method.

Example: $x^3+4x-5=0$. (See Fig. 3.)

The parabola $a^2-c=-4$ cuts the hyperbola $ac=5$ in the point 1, 5. Therefore the real root is 1.

The quadratic of the remaining roots is

$$x^2+x+5=0.$$

This is solved by moving the major axis of the parabola, $a^2=c$, which has been cut out, to the line $x=-\frac{1}{2}$ and letting it cut the y axis at 5. The ordinate of the vertex is the square of the coefficient of i.

The solution is then $-.5 \pm i\sqrt{4.7}=-.5 \pm 2.18i$.

The solution of a cubic in the reduced form

$$x^3+(c-a^2)x-ac=0$$

depends upon ac, and a^2. The product ac may be found on the D scale of a slide rule with a and a^2 on the C and B scales respectively. It is therefore possible to solve a cubic in the above form upon a slide rule.*

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* Runge, "Graphical Methods."